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RAN-2203080301040043

M.Sc. (Sem. - I) Examination November - 2025

Mathematics : Integral Transforms-I (Paper - PGMTH - 1043)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions (1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) Answer all questions (3) Figures to the right indicate marks (4) Follow usual notations and conventions	Seat No.: Student's Signature

Q. 1 Attempt Any Two: **14**

(a) Let $f(t)$ be a periodic function with period ω so that $f(t+n\omega) = f(t)$ $(n \in N)$, then prove that $L[f(t+n\omega)] = \int_0^{\omega} \frac{e^{-st} f(t)}{1-e^{-s\omega}}\,dt$ Find Laplace transform of $f(t) = t^2$, where $f(t+2) = f(t)$.

(b) If $L[f(t)] = \overline{f(s)}$, then prove that $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\overline{f(s)}$

Hence find L[$tsinat$].

(c) (i) Find Laplace Transform of $f(t) = \begin{cases} 2t \; ; & 0 < t < 5 \\ 1 \; : & t > 5 \end{cases}$

(ii) Find $L\left[\frac{sinat}{t}\right]$. Does $L\left[\frac{cosat}{t}\right]$ exist?

Q. 2 Attempt Any Two: **14**

(a) If the function $f(t)$ is piecewise continuous function on every finite interval and it satisfies the condition $|f(t)| \le M e^{at}$, $\forall\; t \ge 0$, for some constants a and M. Prove that Laplace transform of $f(t)$ exists $\forall\; s > a$.

(b) Define finite Laplace transform. Find finite Laplace transform of

(i) $f(t) = cosat$

(ii) $f(t) = t^2$

(c) (i) Using Laplace transform evaluate $\int_0^{\infty} \frac{e^{-t} - e^{-3t}}{t}\,dt$

(ii) Find L[$t^2\,cosat$]

Q. 3 Attempt Any Two: **14**

(a) Find $L^{-1}\left[\frac{1}{(s^2+a^2)(s^2+b^2)}\right]$.

(b) (i) Using convolution theorem evaluate $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$

(ii) Prove that $\int_0^t J_0(u)\,J_0(t-u)du = sint$

(c) (i) Prove that $L^{-1}\left[\tan^{-1}\left(\left(\frac{2}{s^2}\right)\right)\right] = \frac{2}{t}\sin(t)\, sinh(t)$

(ii) Find $L^{-1}\left[\frac{e^{-\pi s}}{s^2+1}\right]$.

Q. 4 **Attempt Any Two:** **14**

(a) Use Laplace transform to solve

$y'' - 5y' + 6y = e^x$, $y(0) = y'(0) = 1$.

(b) Using Laplace transform solve

$u_t + xu_x = x,\ x > 0,\ t > 0$, with the initial and boundary conditions

$u(x, 0) = 0,\ for\ x > 0$, $u(0, t) = 0,\ f$ or $t > 0$

(c) Using Laplace transform, solve the following integral equations

(i) $f(t) = 1 + \int_0^t \sin(t-u)\, f(u)\, du$

(ii) $\int_0^t f(u)\,(t-u)^{-1/3}\, du = t + t^2$

Q. 5 **Attempt Any Two:** **14**

(a) Define convolution of two functions.

If $L^{-1}\ [\overline{f}(s)] = f(t)$ and $L^{-1}\ [\overline{g}(s)] = g(t)$, then prove that

$L^{-1}[\overline{f}(s)\cdot \overline{g}(s)] = (f * g)(t)$

(b) Define Cosine integral function and derive its Laplace transform.

(c) If $L[f(t)] = \overline{f}(s)$, then in usual notations prove that

(i) $L\left[f(at) = \frac{1}{a}\overline{f}\left(\frac{s}{a}\right)\right]$, $a > 0$ is a constant.

(ii) $L[e^{at}f(t)] = \overline{f}(s-a)$.

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